FEM-Source Integral Boundary Conditions for Computation of the Open-Boundary Magnetostatic Fields

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Abstract — A hybrid method, called finite element method-source integral boundary conditions, is proposed for the computation of 3D unbounded magnetostatic fields. The method is based on the evaluation of the vector magnetic potentials on some fictitious boundaries enclosing each part of the magnetic materials, according to the equivalent bound current flowing in their surfaces and volumes or equivalent magnetic dipoles in their volumes. The major advantage of this method lies in its efficiency of computation the magnetostatic fields in which sources and materials are separated in free space by some distances which are large compared to their size.

I. Introduction

Many methods have been devised for the finite element method (FEM) computation of the 3D unbounded magnetostatic fields [1]. Hybrid methods are popular among them. The hybrid methods combine FEM with other methods to solve the unbounded problem [2]. The unbounded problem is divided into two domains by a fictitious boundary. The inner domain is solved by FEM, and the outer domain is solved by other methods. Finite element method-boundary element method (FEM-BEM) is one of the popular hybrid methods in the magnetostatic field. The boundary integral equations in BEM describe the outer region as equivalent surface current and magnetic dipoles, flowing on the fictitious boundary, and calculate the Dirichlet conditions by integral of them at the same time. Because the source points and field points are all on the same boundary, when the distance is close to zero, the calculation error could be great. Finite element method-Dirichlet boundary condition iteration (FEM-DBCI) [3]-[4] is another form of FEM-BEM, because it extended to integral on another boundary, which is surrounded by the fictitious boundary wholly. We proposed a new hybrid method, FEM-source integral boundary conditions. We have used it to calculate electrostatic fields [5]. Now we'll discuss how to use it to solve magnetostatic problems. In the method, the magnetic materials in magnetostatic field are replaced by equivalent bound current, flowing in the surfaces and volumes of the magnetic materials. We calculate the Dirichlet conditions on the fictitious boundary by integral equations.

II. The FEM-Source Integral Boundary Conditions

The unbounded magnetostatic fields are determined by current and materials in the vacuum. After the materials are replaced by equivalent sources, the magnetostatic field can be determined by real currents and equivalent sources in the vacuum.

The FEM-source integral boundary conditions method is illustrated by the simple 3-D magnetostatic system shown in Fig. 1. Fictitious boundaries enclosed several non-homogeneous magnetic materials separately. The method is efficient, Especially when there are some distances among several parts. In the bounded domain D_i, we can take field equation in form of vector magnetic potential $\mathbf{A}$,

$$\frac{1}{\mu}\nabla\times\nabla\times\mathbf{A}=\mathbf{J_c} \tag{1}$$

where μ is the permeability. In FEM, (1) can be discretized as

$$\mathbf{KA}=\mathbf{P} \tag{2}$$

where $\mathbf{K}$ is a sparse matrix and $\mathbf{P}$ is a constant vector column.

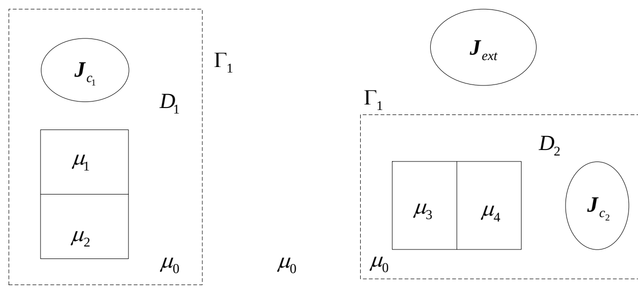

Fig. 1. Fictitious boundaries enclosed a magnetostatic system of non-homogeneous magnetic materials.

To calculate (2) we need boundary conditions on Γ_1. The boundary conditions are expressed by $\mathbf{A}|_{\Gamma_1}$. To calculate the $\mathbf{A}|_{\Gamma_1}$, the materials should be equaled to equivalent sources. The equivalent sources could be either bound current or magnetic dipole. Magnetic materials could be replaced by bound current flowing in its surface and volume or by magnetic dipoles in its volume. The $\mathbf{A}|_{\Gamma_1}$ will be determined by integral of all sources. The sources have external current, equivalent bound current and magnetic dipoles. The integral equations will be discussed in next section.

There are two ways to calculate (2) with the boundary conditions $\mathbf{A}|_{\Gamma_1}$.

One is couple of FEM and integral equations. It can be described as

$$\begin{bmatrix}\mathbf{K_1} & \mathbf{K_2}\\ \mathbf{C_1} & \mathbf{C_2}\end{bmatrix}\begin{bmatrix}\mathbf{A}\\ \mathbf{A}|_{\Gamma_1}\end{bmatrix}=\begin{bmatrix}\mathbf{P_1}\\ \mathbf{P_2}\end{bmatrix} \tag{3}$$

where $\mathbf{K}_1\mathbf{A} + \mathbf{K}_2\mathbf{A}|_{\mathbf{\Gamma}_1} = \mathbf{P_1}$ is a part of (2) without the equations on boundary nodes; $\mathbf{C}_1\mathbf{A} + \mathbf{C}_2\mathbf{A}|_{\mathbf{\Gamma}_1} = \mathbf{P_2}$ is the boundary integral equation.

The other is iteration.

At first, we let

$$\mathbf{A}|_{\mathbf{\Gamma}_1}{}^{(1)} = \mathbf{A}_0{}^{(0)} \tag{4}$$

where $\mathbf{A}_0{}^{(0)}$ is arbitrary constant vector column. It's the initial value of $\mathbf{A}|_{\mathbf{\Gamma}_1}$. Step 2 solves the bounded domains by FEM. Step 3 calculates equivalent bound current density or magnetic dipole volume density of the materials. Step 4 calculates new Dirichlet conditions by boundary integral equation. Step 5 returns to the step 2 with new boundary conditions. We should iterate the problem until it is convergence. The procedure can be described as

$$\begin{bmatrix} \mathbf{K_1} & \mathbf{K_2} \\ \mathbf{0} & \mathbf{1} \end{bmatrix} \begin{bmatrix} \mathbf{A}^{(i)} \\ \mathbf{A}|_{\mathbf{\Gamma}_1}{}^{(i)} \end{bmatrix} = \begin{bmatrix} \mathbf{P_1} \\ \mathbf{A}_0{}^{(i-1)} \end{bmatrix} \quad (i = 1, 2, ..., n) \tag{5}$$

where i indicates the FEM calculation time and $\mathbf{A}|_{\Gamma_1}{}^{(i)}$ is determined by equivalent sources, according $\mathbf{A}^{(i-1)}$.

The iteration method is more efficiency than coupling when the interested problem is complex.

III. Boundary Integral Equations

In magnetostatic fields magnetic materials can either be replaced by equivalent bound current in its surface and volume or be replaced by equivalent magnetic dipoles in its volume.

If the magnetic material is considered as bound current, the surface bound current flow in its surface with a density $\mathbf{K}_M$.

$$\mathbf{K}_M = \mathbf{M} \times \mathbf{e}_n = \frac{\mu - \mu_0}{\mu\mu_0} \nabla \times \mathbf{A} \times \mathbf{e}_n \tag{6}$$

where $\mathbf{M}$ is the magnetization and $\boldsymbol{e}_n$ is a unit vector normal to the material surface. The volume bound current flow in its volume with a density $\mathbf{J}_M$.

$$\mathbf{J}_M = \nabla \times \mathbf{M}\,. \tag{7}$$

If the material is homogeneous, linearity and isotropy, it can be proved that $\mathbf{J}_M = 0$.

Therefore, the Dirichlet boundary conditions can be calculated by integral equation, that is

$$\begin{aligned} \mathbf{A}|_{\Gamma_1} &= \frac{\mu_0}{4\pi} \iiint \frac{\mathbf{J}_c}{R_1} dV_1' + \frac{\mu_0}{4\pi} \iiint \frac{\mathbf{J}_{ext}}{R_2} dV_2' \\ &+ \frac{\mu_0}{4\pi} \oiint \frac{\mathbf{K}_M}{R_3} dS_3' \end{aligned} \,. \tag{8}$$

If the magnetic material is considered as magnetic dipoles in its volume, its contribution to Dirichlet boundary conditions can be calculated by $\mathbf{M}$ directly. The boundary integral equation is

$$\begin{aligned} \mathbf{A}|_{\Gamma_1} &= \frac{\mu_0}{4\pi} \iiint \frac{\mathbf{J}_c}{R_1} dV_1' + \frac{\mu_0}{4\pi} \iiint \frac{\mathbf{J}_{ext}}{R_2} dV_2' \\ &+ \frac{\mu_0}{4\pi} \iiint \frac{\mathbf{M} \times \mathbf{e}_{R_4}}{R_4{}^2} dV_3' \end{aligned} \,. \tag{9}$$

IV. Validation Examples

In this example, we computed the magnetic flux density above a thin plate of permeability μ. There is a conductor with volume current density $\mathbf{J}_c$ under the plate. The conductor and thin plate are all embedded in free space, as shown in Fig. 2. Through the example, we can know the shielding effect of steady magnetic field.

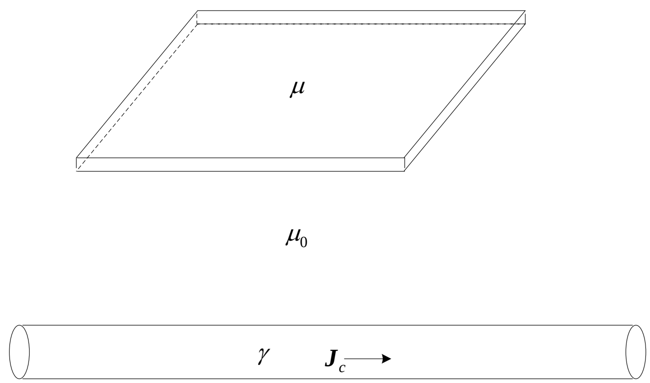

Fig. 2. A thin plate with the presence of volume current.

V. Conclusion

The FEM-source integral boundary conditions method is mainly intended to change unbounded static problems to bounded problems by adding an accurate Dirichlet conditions to FEM. It could also be used in time-varying fields. When using the method, few critical problems need to be solved. The first is how to converge quickly. The second is the study of rapid integral method and the third is how to improve the accuracy of the integral equation.

VI. References

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